

Algebra Refresher Workshop: Exponent Rules

1. $a^n = a \cdot a \cdot a \cdots a$ (n times) Example: $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$

2. $(ab)^n = a^n b^n$

Why? Example: $(2 \cdot 5)^3 = (2 \cdot 5)(2 \cdot 5)(2 \cdot 5) = (2 \cdot 2 \cdot 2)(5 \cdot 5 \cdot 5) = 2^3 \cdot 5^3$

3. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Why? Example: $\left(\frac{3}{2}\right)^4 = \left(\frac{3}{2}\right)\left(\frac{3}{2}\right)\left(\frac{3}{2}\right)\left(\frac{3}{2}\right) = \frac{3 \cdot 3 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 2} = \frac{3^4}{2^4}$

4. $a^m \cdot a^n = a^{m+n}$

Why? Example: $2^3 \cdot 2^4 = (2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2) = (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) = 2^7$ (which is 2^{3+4})

5. $\frac{a^m}{a^n} = a^{m-n}$

Why? Example: $\frac{4^5}{4^3} = \frac{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}{4 \cdot 4 \cdot 4} = \frac{4 \cdot \cancel{4} \cdot \cancel{4} \cdot \cancel{4}}{\cancel{4} \cdot \cancel{4} \cdot \cancel{4}} = \frac{4 \cdot 4}{1} = 4 \cdot 4 = 4^2$ (which is 4^{5-3})

6. $a^{-n} = \frac{1}{a^n}$

Why? Example: on one hand... $\frac{8^2}{8^5} = \frac{8 \cdot 8}{8 \cdot 8 \cdot 8 \cdot 8 \cdot 8} = \frac{\cancel{8} \cdot \cancel{8}}{8 \cdot 8 \cdot \cancel{8} \cdot \cancel{8}} = \frac{1}{8 \cdot 8} = \frac{1}{8^3}$
on the other hand... $\frac{8^2}{8^5} = 8^{2-5} = 8^{-3}$, so $\frac{1}{8^3} = 8^{-3}$

7. $a^0 = 1$ (as long as $a \neq 0$)

Why? Example: on one hand... $\frac{4^3}{4^3} = 4^{3-3} = 4^0$
on the other hand... $\frac{4^3}{4^3} = \frac{4 \cdot 4 \cdot 4}{4 \cdot 4 \cdot 4} = \frac{\cancel{4} \cdot \cancel{4} \cdot \cancel{4}}{\cancel{4} \cdot \cancel{4} \cdot \cancel{4}} = \frac{1}{1} = 1$, so $4^0 = 1$

8. $(a^m)^n = a^{m \cdot n}$

Why? Example: $(5^3)^2 = (5^3)(5^3) = 5^{3+3} = 5^6$ (which is $5^{3 \cdot 2}$)

9. $a^{1/n} = \sqrt[n]{a}$

Examples: $5^{1/2} = \sqrt{5}$ and $8^{1/6} = \sqrt[6]{8}$

10. $a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

Why? Example: $7^{4/5} = 7^{4 \cdot 1/5} = (7^4)^{1/5} = \sqrt[5]{7^4}$
or $= (7^{1/5})^4 = (\sqrt[5]{7})^4$

Why does $\sqrt[n]{a^m} = (\sqrt[n]{a})^m$? Ex: $8^{2/3} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$ or $8^{2/3} = (\sqrt[3]{8})^2 = (2)^2 = 4$

Make up your own numerical examples for the rules that still seem fuzzy or suspicious!